CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Steps to Solving Equations

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley
Beta Version

Steps to Solving Equations

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Form and solve linear equations involving factorizing and using the distributive law.

In particular, this unit aims to help you identify and assist students who have difficulties in:

- Using variables to represent quantities in a real-world or mathematical problem.
- Solving word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

7.EE: Use properties of operations to generate equivalent expressions.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.

INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson, students attempt the assessment task individually. You then review students' responses and formulate questions that will help them improve their work.
- During the lesson, students work collaboratively in pairs or threes, matching equations to stories and then ordering the steps used to solve these equations. Throughout their work, students explain their reasoning to their peers.
- Finally, students again work individually to review their work and attempt a second task, similar to the initial assessment task.

MATERIALS REQUIRED

- Each student will need copies of the assessment tasks *Express Yourself* and *Express Yourself (revisited)*, and *Card Set: Stories* (not cut up), a mini-whiteboard, a pen, and an eraser.
- For each small group of students provide cut up copies of *Card Set: Stories* (cut up), *Card Set: Equations*, and *Card Set: Steps to Solving*, a large sheet of paper for making a poster, a marker, and a glue stick.
- There are also some projector resources to help with whole-class discussion.

TIME NEEDED

15 minutes before the lesson for the assessment task, a 1-hour lesson, and 15 minutes in a follow-up lesson (or for homework). All timings are approximate, depending on the needs of your students.

BEFORE THE LESSON

Assessment task: *Express Yourself* (15 minutes)

Have the students do this task, in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and identify students who have misconceptions or need other forms of help. You should then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of *Express Yourself*. Introduce the task briefly and help the class to understand what they are being asked to do.

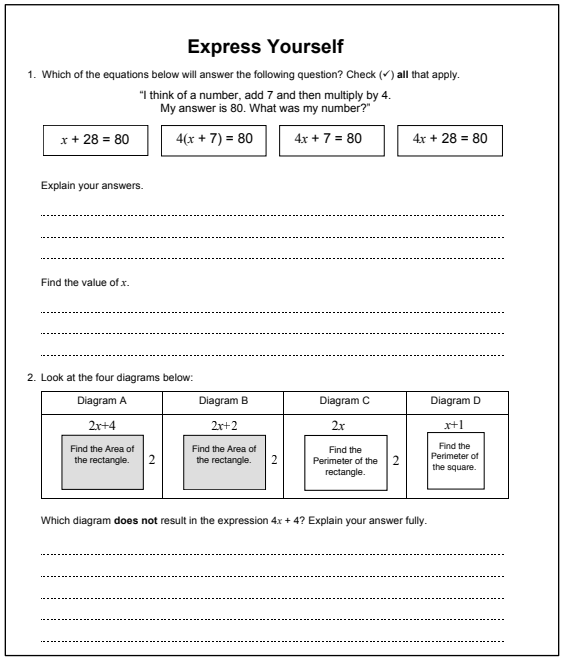
Spend 15 minutes working individually, answering these questions.

Show all your work on the sheet.

Make sure you explain your answers really clearly.

It is important that, as far as possible, students answer the questions without assistance.

Students should not worry too much if they cannot understand or do everything because you will teach a lesson using a similar task, which should help them. Explain to students that, by the end of the next lesson, they should expect to answer questions such as these. This is their goal.



Express Yourself

1. Which of the equations below will answer the following question? Check (✓) **all** that apply.

"I think of a number, add 7 and then multiply by 4.
My answer is 80. What was my number?"

$x + 28 = 80$	$4(x + 7) = 80$	$4x + 7 = 80$	$4x + 28 = 80$

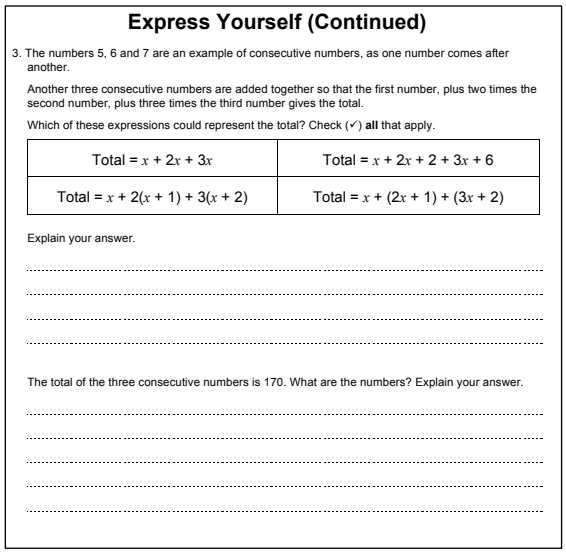
Explain your answers.

Find the value of x.

2. Look at the four diagrams below:

Diagram A	Diagram B	Diagram C	Diagram D
$2x+4$ Find the Area of the rectangle. 2	$2x+2$ Find the Area of the rectangle. 2	$2x$ Find the Perimeter of the rectangle. 2	$x+1$ Find the Perimeter of the square.

Which diagram **does not** result in the expression $4x + 4$? Explain your answer fully.



Express Yourself (Continued)

3. The numbers 5, 6 and 7 are an example of consecutive numbers, as one number comes after another.

Another three consecutive numbers are added together so that the first number, plus two times the second number, plus three times the third number gives the total.

Which of these expressions could represent the total? Check (✓) **all** that apply.

Total = $x + 2x + 3x$	Total = $x + 2x + 2 + 3x + 6$
Total = $x + 2(x + 1) + 3(x + 2)$	Total = $x + (2x + 1) + (3x + 2)$

Explain your answer.

The total of the three consecutive numbers is 170. What are the numbers? Explain your answer.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and difficulties. The purpose of this is to forewarn you of the issues that will arise during the lesson, so that you may prepare carefully.

We suggest that you do not score students' work. Research shows that this is counterproductive as it encourages students to compare scores, and distracts their attention from how they may improve their mathematics.

Instead, help students to make further progress by asking questions that focus attention on aspects of their work. Some suggestions for these are given in the table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We strongly recommend that you write your own lists of questions, based on your students' work, using the ideas in the *Common issues* table. You may choose to write questions on each student's work. If you do not have time for this, then prepare a few questions that apply to most students and write these on the board when the assessment task is revisited.

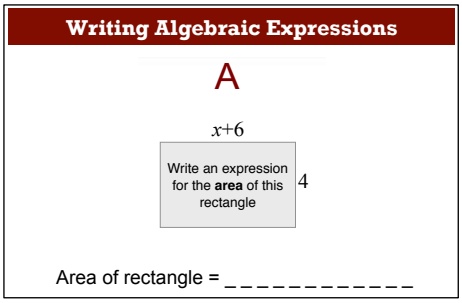
Common issues:	Suggested questions and prompts:
Student applies operations in the wrong order (Q1) For example: The student chooses $4x + 7 = 80$ as an appropriate equation. Or: The student chooses $x + 28 = 80$ as an appropriate equation.	• In this expression, what is the first thing that happens to the number I am thinking of? Then what happens? • What does x represent? What are you adding 7 to? • Is adding 7 and then multiplying by 4 the same as adding 28? How could you check this?
Student does not recognize all relevant expressions (Q1) For example: The student chooses $4(x + 7) = 80$ as the only appropriate equation.	• How else could you write the expression $4(x + 7)$?
Student does not distinguish between area and perimeter (Q2) For example: The student writes an expression for the area instead of the perimeter of the rectangles in Diagrams C and D. Or: The student writes an expression for the perimeter instead of the area of the rectangles in Diagrams A and B.	• How do you calculate the area of a rectangle? • What does perimeter mean? • Does your expression represent the area or the perimeter of this rectangle?
Student assumes the three numbers are equal (Q3) For example: The student selects Total = $x + 2x + 3x$ as an appropriate expression.	• What does 'consecutive' mean? • What does x represent? • Can you try some numbers to check that this works?
Student does not multiply all terms in the bracket (Q3) For example: The student selects Total = $x + (2x + 1) + (3x + 2)$ as an appropriate expression.	• What does x represent? • How do you write 'one more than x' using algebra? Now read the question again: what happens next? What happens if you add two of these numbers together?
Student calculates an incorrect value for x (Q1, Q3)	• If you substitute your value of x into the left hand side of the equation, does it equal the number on the right hand side? • How will you check whether your value for x is correct?
Student does not interpret the solution For example: The student does not realize that x represents the number first thought of (Q1). Or: The student does not recognize that $x = 27$ is the first of the three consecutive numbers found (Q3).	• You have found that $x = 27$. Read the question again. What are the **three** consecutive numbers?
Student completes the task	• Can you make up a situation that would lead to the equation $4(x + 3) = 16$? • Could you solve these equations using a different method? What would the method be?

SUGGESTED LESSON OUTLINE

Whole-class introduction: (10 minutes)

Give each student a mini-whiteboard, pen, and an eraser.

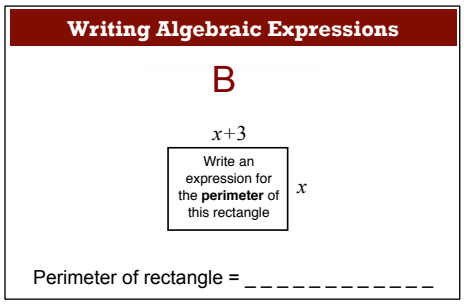
Display Slide P-1 of the projector resource.



Write an expression for the area of this rectangle on your whiteboard.

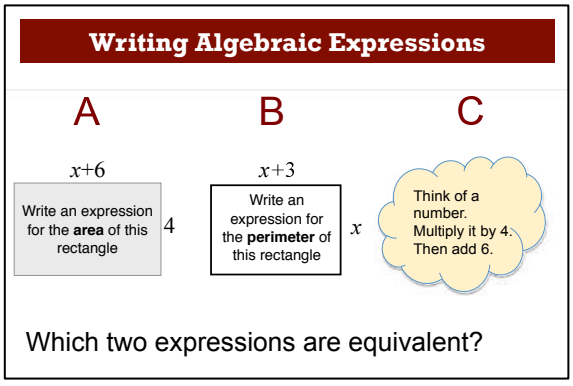
Spend time discussing the expressions students give. Some students may write the expression $4(x + 6)$ whereas others may apply the distributive law to give $4x + 24$. Explain their equivalence by considering how the area of the single rectangle $4(x+6)$ may be split into the two smaller areas $4x$ and 24 by drawing a vertical line. Notice whether students make the mistake of writing the expression as $4x + 6$ or whether they confuse the area of the rectangle with the perimeter.

Display Slide P-2 of the projector resource:



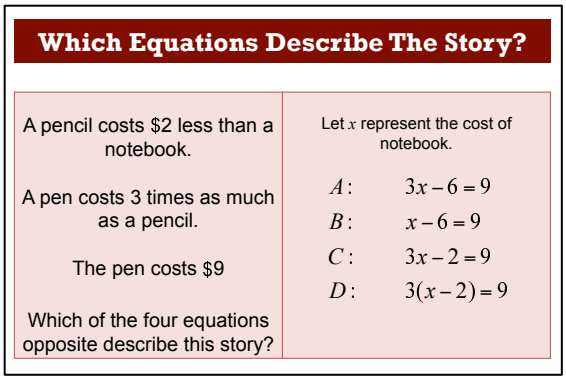
Write an expression for the perimeter of this rectangle on your whiteboard.

Again, spend time discussing the expressions given by students. Notice whether students collect like terms to give $2(2x + 3)$ or $4x + 6$, or whether they give an un-simplified expression, for example, $x + 3 + x + x + 3 + x$. Display Slide P-3 of the projector resource:



Ask students to compare the expressions they have written for A and B with the expression that arises from the description in C. Students should be able to identify that $4x + 6$ is a suitable expression for C and so the expressions for B and C are the same.

Display Slide P-4 of the projector resource:



Students will often look at the numbers contained within an expression/equation when matching it with a story and, as a result, misinterpret the description given.

Write the equations that you think represent the story on your whiteboard.

Students should be encouraged to think carefully about this and explore the differences between the four equations. Discuss the responses given and spend some time discussing why equations A and D are correct and why the others are incorrect:

If x is the cost of a notebook, what expression will give the cost of a pencil? [x-2]

If a pen costs 3 times as much as a pencil, what expression will give the cost of a pen? [3(x-2) or 3x-6]

What mistakes have been made with B and C?
[The expression x-2 has been multiplied by 3 incorrectly in both cases.]

OK, so what is the cost of the notebook? [$5]. Can we check that this fits our equations?

Explain to students that in the next activity they will be writing and matching equations to stories in a similar way.

Individual work: Writing equations (5 minutes)

Give each student *Card Set: Stories* (not cut up).

Here are six stories.

Spend 10 minutes on your own writing an equation for each of the stories.

In each case, let x represent the number you are trying to find.

Do not worry if you can't write an equation for every story as, later on, you will be working in groups on this.

In the next activity, students will be given six equations to match up with these stories; some of these may have been simplified or written in a different form. This individual work should, therefore, help students with the matching process as well as providing an opportunity for them to think carefully about the equations, and look beyond the surface features.

Collaborative activity: Matching cards (10 minutes)

Organize students into groups of two or three.

For each group provide a cut-up copy of the *Card Set: Stories* and *Card Set: Equations*.

> *The six story cards are the same stories as you have just been looking at.*
>
> *Working together in your group, your job is to match each story with an equation.*
>
> *Use the work you have done individually to help you.*
>
> *Check to see whether any of the equations you have written down match the equations on the cards.*

It is likely that students who have identified correct equations may have written them in a different form to the equations on the card. Encourage them to check whether what they have written is the same. Some students may have an incorrect equation, but assume it is correct. Encourage students to check their work carefully.

While students work in small groups you have two tasks: to make a note of students approaches to the task, and to support student reasoning.

Make a note of student approaches to the task

Listen and watch students carefully. Note any common errors in algebra and computation. Do students use the distributive law correctly? Do they only multiply part of an expression? Notice the ways students check to see if their card-match is correct. Do they substitute back into the equations? Do they know which value x represents?

Notice the quality and depth of students' explanations. Are students satisfied just to match the cards, or do they explain choices? Do they challenge each other if they disagree on a matched pair?

Support student reasoning

Prompt students to explain what expressions mean clearly.

> *What does x represent in this story?*
> *What information do you have? What do you need to find out?*
>
> *You've decided how you're going to write [how old James is/the score for Paper 1/the cost of a strawberry chew]. What's the connection between [his age and his dad's age/the scores/the costs of a chew and a lollipop]? How does that help you to write [his dad's age/the score for paper 2/the cost of a lollipop]?*

Encourage students to explain their reasoning carefully, and check that all group members are able to justify each choice.

> *Jean, you matched these cards. Terry, do you agree that these cards match? Explain please.*

If students finish quickly, ask them to write their own, different stories to match the equation cards.

Sharing work (5 minutes)

As students finish matching the cards, ask them to jot down the matched pairs on their whiteboards (for example, S1 with E1, S5 with E2, etc.). Then ask one student from each group to visit another group. This way they can compare their own matches with another group's.

The student remaining at the desks is to explain their reasoning for the matched cards to the visiting student.

Students may now want to make changes or additions to their matches, especially if they have visited a group that has matched up different stories to their own. If this is the case, it is important that

students are able to explain the new match. They should not just assume that another group's matches are correct without exploring the reasoning used.

Whole-class discussion (5 minutes)

It is likely that some groups may not have managed to match all six stories with an equation. Spend a few minutes discussing some of the matches the students have made. Survey the students to see if, after sharing their work with another group, they have changed their mind. Ask them to explain and justify their reasoning.

Harry, which equation did you match with S3? How did you decide?

Did anyone match a different equation with this story? Explain your thinking.

Which equation is the correct match?

Did any group change their mind about a match? Which story/equation was it? What did you think it was originally? What did you change it to? Explain why you did this.

The aim of this discussion is explore the reasoning behind some of the matches and help students to justify their thinking, not to check that all groups have successfully matched all of the cards.

Collaborative activity: Posters that show steps to solving four equations (20 minutes)

Give each group of students a large piece of poster paper, a marker, and a glue stick.

Put the cards E5 and E6 and the story cards you've matched with them to one side.
Divide your large sheet of paper into quarters.

You are now going to work with equation cards E1 – E4.
Stick one at the top of each section, along with the matched-story.
If you haven't managed to match all four of the equation cards with a story yet, just stick down the four equation cards.

Students don't need to stick the last two sets of cards in place as they are not used in the second matching activity. Nevertheless, if the sheets of paper you have provided are very large, they may wish to do this.

For each group provide a cut-up copy of *Card Set: Steps to Solving*.

You are going to explore the steps to solving these four equations.

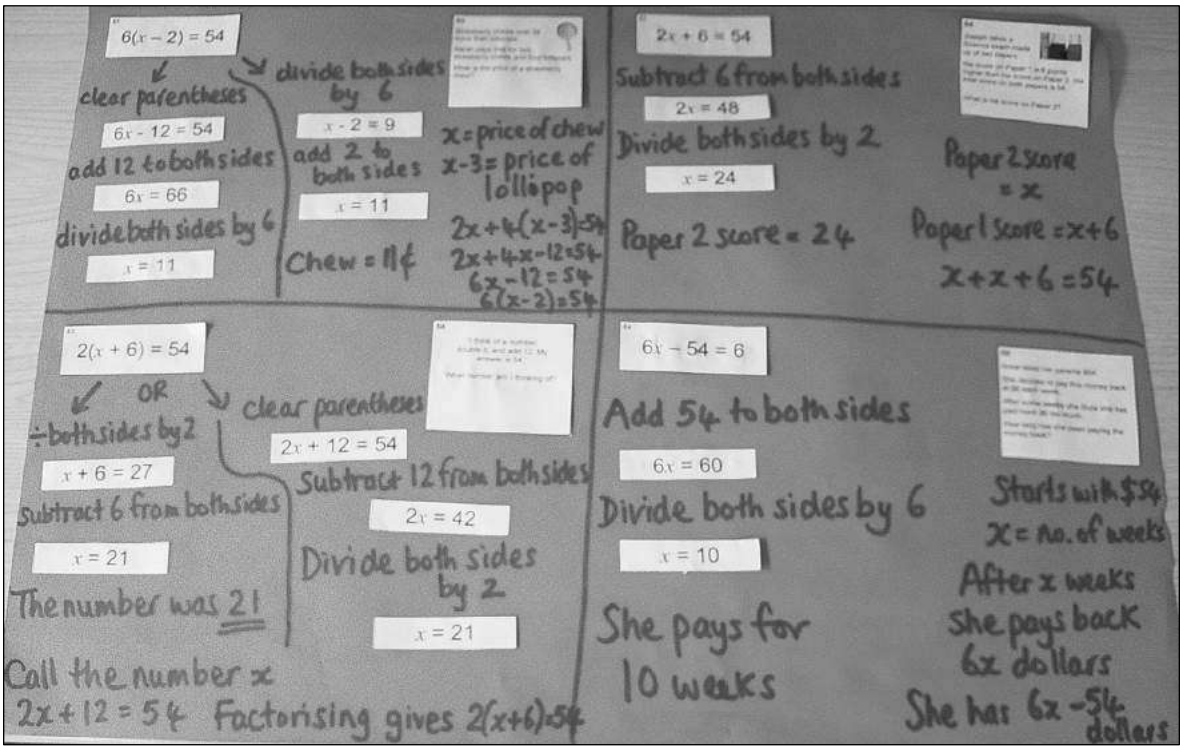
In between each step write a description of the process involved. For example, you may write something like 'divide both sides by 2' or 'add 6 to both sides'. Repeat this until you finally reach a solution.

If you find there is more than one method for solving an equation, stick the two solutions side-by-side.

Once students have completed this work, they can finish any matching of pairs. Then encourage them to add explanations to their posters to show how they arrived at an equation for each of their chosen stories.

As students work, support them as before. Walk around, watch, and listen, and check that students are writing a description for each step of the solution process.

The finished poster may look something like this:



Whole-class discussion (10 minutes)

Select two or three students from different groups that have completed a solution for *Equation Cards E1* and/or *E3*. Ask them to explain why there are two methods for solving these equations.

Which of the two methods is the most efficient?

Which method do you prefer? Why?

Is there a different method that could be used to solve these equations?

Students may prefer to clear parentheses, even though this creates an extra step in the solution process.

*What do you need to remember when using the distributive property to clear parentheses? [To multiply **every** term by the term outside.]*

How else could we clear parentheses? [e.g. 2(x + 3) = (x + 3) +(x + 3) = x + 3 + x + 3.]

The focus of this discussion is to explore the processes involved in a range of different approaches, not to promote a particular method.

Follow-up lesson: Review individual solutions to the assessment task (15 minutes)

Give students their responses to the original assessment task, *Express Yourself,* and a copy of the task *Express Yourself (revisited*). Some teachers set this task as homework.

If you have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only the questions they think are appropriate to their own work.

Look at your written script and think about what you have learned since you did this task.

Make some notes on what you have learned during the lesson.

Now have a go at the second sheet, Express Yourself (revisited*). Can you use what you have learned to answer these questions?*

SOLUTIONS

Assessment task: *Express Yourself*

1. The task is to write this sentence using algebra: "I think of a number, add 7 and multiply by 4. My answer is 80." $4(x + 7) = 80$ and $4x + 28 = 80$ are two ways of representing this.

   Students choosing the other two equations may have not applied the distributive property to one of the terms in the left-hand expression.

   $x = 13$ represents the number I was thinking of.

2. Diagram A does not describe the algebraic expression $4x + 4$. The area of this rectangle is $4x + 8$.

3. Total $= x + 2x + 2 + 3x + 6$ and Total $= x + 2(x + 1) + 3(x + 2)$ are the expressions that match the sentence.

   Students may simplify the expression, before solving the equation:

   $6x + 8 = 170$
   $6x = 162$
   $x = 27$.

   The consecutive numbers are 27, 28 and 29.

Lesson task

In the first card matching activity, these are the correct pairs:

S1 → E5.
S2 → E6.
S3 → E1.
S4 → E2.
S5 → E4.
S6 → E3.

These are the matches that provide the 'steps to solving' the Equations on Cards E1 to E4:

E1 $6(x – 2) = 54$

Method 1: $6(x – 2) = 54$
Divide both sides by 6
$x – 2 = 9$
Add 2 to both sides
$x = 11$.

Method 2: $6(x – 2) = 54$
Multiply out the brackets
$6x – 12 = 54$
Add 12 to both sides
$6x = 66$
Divide both sides by *6*
$x = 11$.

A strawberry chew costs 11¢ (and a lollipop costs 8¢).

E2 $2x + 6 = 54$

$2x + 6 = 54$
Subtract 6 from both sides
$2x = 48$
Divide both sides by 2
$x = 24$.

The score for Paper 2 was 24 marks.

E3 $2(x + 6) = 54$

Method 1: *$2(x + 6) = 54$*
Multiply out the brackets
$2x + 12 = 54$
Subtract 12 from both sides
$2x = 42$
Divide both sides by 2
$x = 21$.

Method 2: $2(x + 6) = 54$
Divide both sides by 2
$x + 6 = 27$
Subtract 6 from both sides
$x = 21$.

The number I was thinking of was 21.

E4 $6x - 54 = 6$

$6x - 54 = 6$
Add 54 to both sides
$6x = 60$
Divide both sides by 6
$x = 10$.

She has been paying for 10 weeks

Assessment task: *Express Yourself (revisited)*

1. Alicia's statement can be represented by these equations:

   $3x + 24 = 66$, and $3(x + 8) = 66$.

   Students choosing the other two equations may have not applied the distributive property to one of the terms in the left-hand expression.

   x represents the number Alicia first thought of.

   The method used to solve the equation will depend on which representation the student chooses to work with. The correct solution is $x = 14$.

2. Diagram A: the perimeter is $2((2x + 1) + 3) = 2(2x + 4) = 4x + 8$.

   Diagram B: the perimeter is $2((2x + 2) + 3)=2(2x + 5)=4x + 10$.

   Diagram C: the area is $2(x + 2) \times 2 = 4 (x + 2) = 4x + 8$.

   Diagram D: the perimeter of the rectangle is $2((x + 3) + (x + 1)) = 2(2x + 4) = 4x + 8$.

   Thus the diagrams that match the expression are A, C, and D.

3. The expressions for three consecutive numbers are $x, x + 1, x + 2$.

   Total = $3x + 3\ x + 3 + 3\ x + 6$ and Total = $3\ x + 3(x + 1) + 3(x + 2)$ are the expressions that match the sentence.

   Summing the terms gives $x + x + 1 + x + 2$. Students may simplify this before or after multiplying by three:

   $3(x + x + 1 + x + 2) = 162$ or $3(3x + 3) = 162$

   $3x + 3x + 3 + 3x + 6 = 162$ or $9x + 9 = 162$

   $9x + 9 = 162$

   From this point, the solution methods are the same.

   $9x = 153$; $x = 17$.

   The consecutive numbers are 17, 18, 19.

Express Yourself

1. Which of the equations below will answer the following question? Check (✓) **all** that apply.

"I think of a number, add 7 and then multiply by 4.
My answer is 80. What was my number?"

$x + 28 = 80$	$4(x + 7) = 80$	$4x + 7 = 80$	$4x + 28 = 80$

Explain your answers.

Find the value of x.

2. Look at the four diagrams below:

Diagram A	Diagram B	Diagram C	Diagram D
$2x+4$ Find the Area of the rectangle. 2	$2x+2$ Find the Area of the rectangle. 2	$2x$ Find the Perimeter of the rectangle. 2	$x+1$ Find the Perimeter of the square.

Which diagram **does not** result in the expression $4x + 4$? Explain your answer fully.

Express Yourself (continued)

3. The numbers 5, 6 and 7 are an example of consecutive numbers, as one number comes after another.

   Another three consecutive numbers are added together so that the first number, plus two times the second number, plus three times the third number gives the total.

   Which of these expressions could represent the total? Check (✓) **all** that apply.

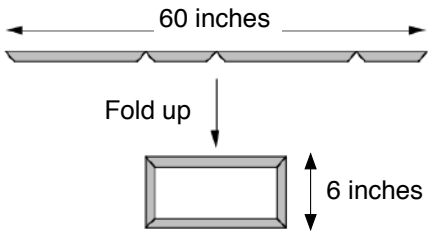
Total = $x + 2x + 3x$	Total = $x + 2x + 2 + 3x + 6$
Total = $x + 2(x + 1) + 3(x + 2)$	Total = $x + (2x + 1) + (3x + 2)$

   Explain your answer.

   The total of the three consecutive numbers is 170. What are the numbers? Explain your answer.

Card Set: Stories

S1



60 inches of plastic are folded to make a picture frame.

The height of the finished frame is 6 inches. How long is the frame?

S2

Tom is 57 years old.

Tom has a son called James.

In three years time Tom will be twice as old as James.

How old is James?

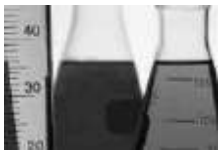
S3

Strawberry chews cost 3¢ more than lollipops.

Sarah pays 54¢ for two strawberry chews and four lollipops.

What is the price of a strawberry chew?

S4



Joseph takes a Science exam made up of two papers.

His score on Paper 1 is 6 points higher than his score on Paper 2. His total score on both papers is 54.

What is his score on Paper 2?

S5

Anna owes her parents $54.

She decides to pay this money back at $6 each week.

After some weeks she finds she has paid back $6 too much.

How long has she been paying the money back?

S6

"I think of a number, double it, and add 12. My answer is 54."

What number am I thinking of?

Card Set: Equations

E1 $6(x - 2) = 54$	**E2** $2x + 6 = 54$
E3 $2(x + 6) = 54$	**E4** $6x - 54 = 6$
E5 $2x + 12 = 60$	**E6** $2(x + 3) = 60$

Card Set: Steps to Solving

$6x = 60$	$2x = 48$
$6x = 66$	$x - 2 = 9$
$x = 24$	$x + 6 = 27$
$x = 11$	$x = 21$
$6x - 12 = 54$	$2x = 42$
$x = 10$	$x = 11$
$2x + 12 = 54$	$x = 21$

Express Yourself (revisited)

1. Which of the equations below will answer the following question? Check (✓) **all** that apply.

"I think of a number, add 8 and then multiply by 3.
My answer is 66. What was my number?"

$x + 24 = 66$	$3x + 8 = 66$	$3x + 24 = 66$	$3(x + 8) = 66$

Explain your answers.

Find the value of x.

2. Look at the four diagrams below:

Diagram A	Diagram B	Diagram C	Diagram D
$2x+1$	$2(x+1)$	$2(x+2)$	$x+3$
Find the Perimeter of the rectangle. 3	Find the Perimeter of the rectangle. 3	Find the Area of the rectangle. 2	Find the Perimeter of the rectangle. $x+1$

Check (✓) **every** diagram that represents the expression $4x + 8$:

Explain your answers.

Express Yourself (revisited) (continued)

3. Three consecutive numbers are added together and then their sum is multiplied by three.

   Some of the equations below represent the total using algebra. Check (✓) **all** that apply.

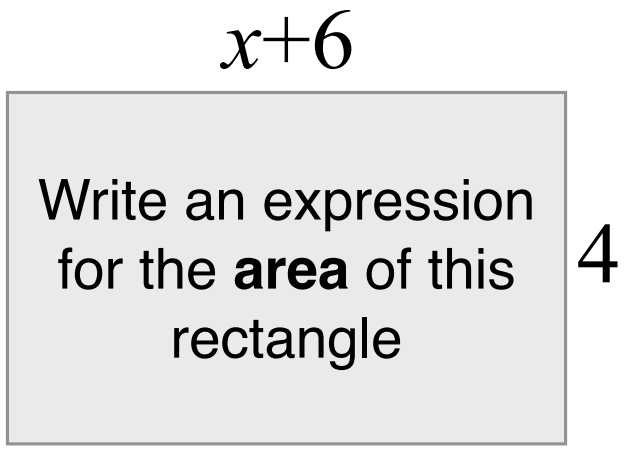
Total = $3x + 3x + 1 + 3x + 2$	Total = $3x + 3x + 3 + 3x + 6$
Total = $3x + 3(x + 1) + 3(x+2)$	Total = $x + x + 3 + x + 6$

   Explain your answers.

   The total of the equation is 162. What are the three consecutive numbers? Explain your answer.

Writing Algebraic Expressions

A



Area of rectangle = _ _ _ _ _ _ _ _ _ _ _ _ _ _

Writing Algebraic Expressions

B

$x+3$

Write an expression for the **perimeter** of this rectangle

Perimeter of rectangle = _ _ _ _ _ _ _ _ _ _ _ _ _ _

Writing Algebraic Expressions

A

$x+6$

Write an expression for the **area** of this rectangle

4

B

$x+3$

Write an expression for the **perimeter** of this rectangle

x

C

Think of a number.
Multiply it by 4.
Then add 6.

Which two expressions are equivalent?

Which Equations Describe The Story?

A pencil costs \$2 less than a notebook.

A pen costs 3 times as much as a pencil.

The pen costs \$9

Which of the four equations opposite describe this story?

Let x represent the cost of notebook.

A: $3x - 6 = 9$

B: $x - 6 = 9$

C: $3x - 2 = 9$

D: $3(x - 2) = 9$

Mathematics Assessment Project

CLASSROOM CHALLENGES

This lesson was designed and developed by the
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