Relational Model and Algebra

Introduction to Databases

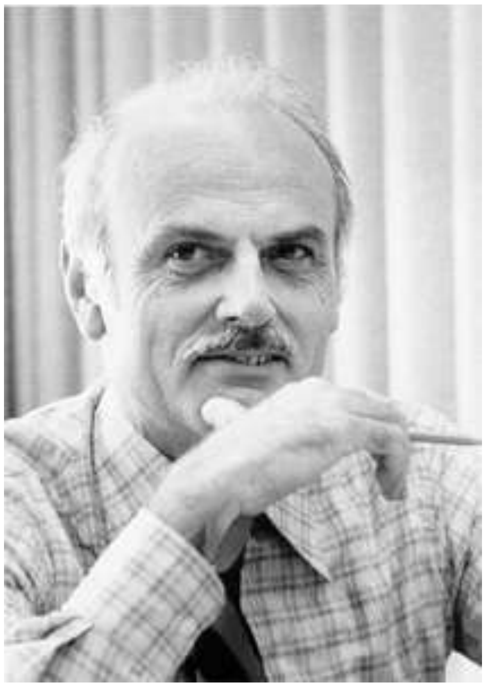
CompSci 316 Fall 2017

DUKE
COMPUTER SCIENCE

Announcements (Tue. Sep. 4)

- Registration: class size will stay at 140; as a courtesy to others, please add/drop ASAP
- Homework #1 to be posted today; due in 2 weeks
  - Sign up for Piazza & Gradiance
  - Set up VM (instructions on course website)
- TA/UTA office hours to be posted soon

Edgar F. Codd (1923-2003)



- Pilot in the Royal Air Force in WW2
- Inventor of the relational model and algebra while at IBM
- Turing Award, 1981

http://en.wikipedia.org/wiki/File:Edgar_F_Codd.jpg

Relational data model

- A database is a collection of relations (or tables)
- Each relation has a set of attributes (or columns)
- Each attribute has a name and a domain (or type)
  - Set-valued attributes are not allowed
- Each relation contains a set of tuples (or rows)
  - Each tuple has a value for each attribute of the relation
  - Duplicate tuples are not allowed
    - Two tuples are duplicates if they agree on all attributes

☞Simplicity is a virtue!

Example

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

Group

gid	*name*
abc	Book Club
gov	Student Government
dps	Dead Putting Society
…	…

Member

uid	*gid*
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
…	…

Ordering of rows doesn't matter (even though output is always in some order)

Schema vs. instance

- Schema (metadata)
  - Specifies the logical structure of data
  - Is defined at setup time
  - Rarely changes
- Instance
  - Represents the data content
  - Changes rapidly, but always conforms to the schema

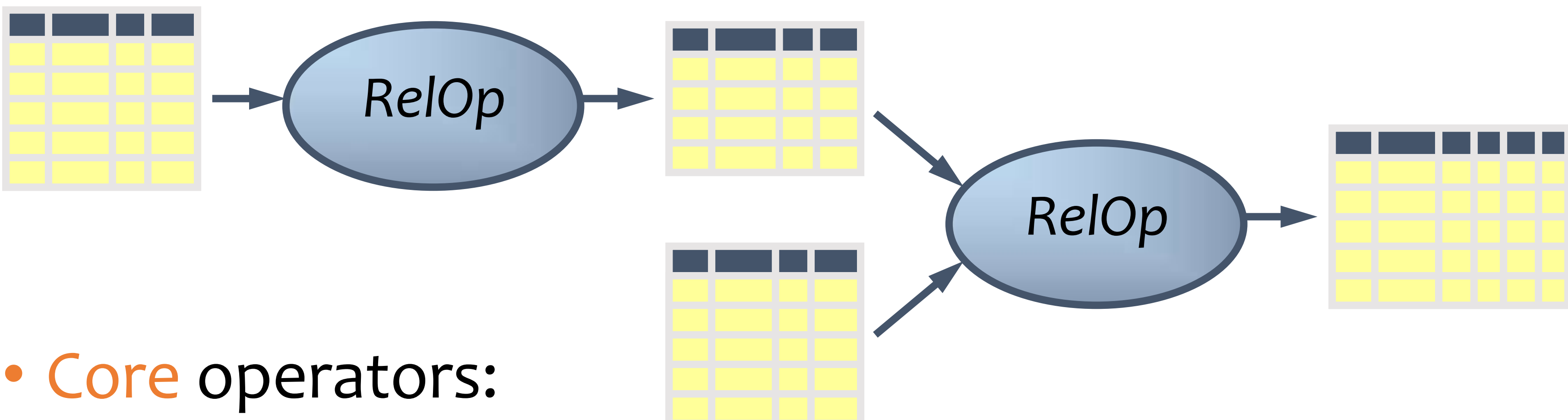
☞Compare to types vs. collections of objects of these types in a programming language

Example

- Schema
  - *User* (*uid* int, *name* string, *age* int, *pop* float)
  - *Group* (*gid* string, *name* string)
  - *Member* (*uid* int, *gid* string)
- Instance
  - *User*: {⟨`142`, `Bart`, `10`, `0.9`⟩, ⟨`857`, `Milhouse`, `10`, `0.2`⟩, …}
  - *Group*: {⟨`abc`, `Book Club`⟩, ⟨`gov`, `Student Government`⟩, …}
  - *Member*: {⟨`142`, `dps`⟩, ⟨`123`, `gov`⟩, …}

Relational algebra

A language for querying relational data based on "operators"



- Core operators:
  - Selection, projection, cross product, union, difference, and renaming
- Additional, derived operators:
  - Join, natural join, intersection, etc.
- Compose operators to make complex queries

Selection

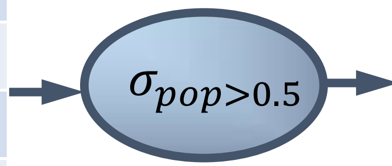
- Input: a table R
- Notation: $\sigma_p R$
  - p is called a selection condition (or predicate)
- Purpose: filter rows according to some criteria
- Output: same columns as R, but only rows or R that satisfy p

Selection example

- Users with popularity higher than 0.5

$$\sigma_{pop>0.5} User$$

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…



uid	*name*	*age*	*pop*
142	Bart	10	0.9
[redacted]			
857	Lisa	8	0.7
[redacted]			
…	…	…	…

More on selection

- Selection condition can include any column of R, constants, comparison ($=$, $\leq$, etc.) and Boolean connectives ($\wedge$: and, $\vee$: or, $\neg$: not)
  - Example: users with popularity at least 0.9 and age under 10 or above 12

$$\sigma_{pop \geq 0.9 \wedge (age < 10 \vee age > 12)}\ User$$

- You must be able to evaluate the condition over each single row of the input table!
  - Example: the most popular user

$$\sigma_{pop \geq every\ pop\ in\ User}\ User$$

WRONG!

Projection

- Input: a table R
- Notation: $\pi_L R$
  - L is a list of columns in R
- Purpose: output chosen columns
- Output: same rows, but only the columns in L

Projection example

- IDs and names of all users

$$\pi_{uid,name}\ User$$

uid	name	age	pop
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

$\pi_{uid,name}$

uid	name
142	Bart
123	Milhouse
857	Lisa
456	Ralph
…	…

More on projection

- Duplicate output rows are removed (by definition)
  - Example: user ages

$$\pi_{age}\, User$$

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

age
10
[redacted]
8
[redacted]
…

Cross product

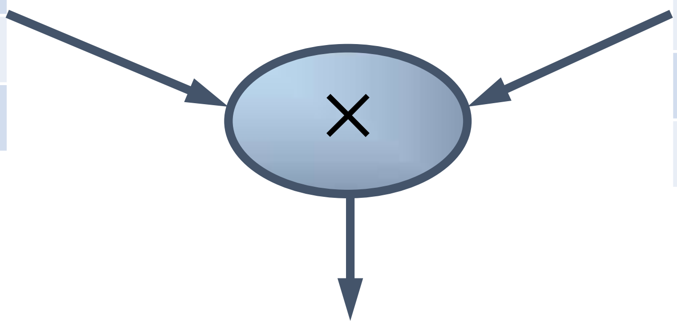
- Input: two tables R and S
- Natation: $R \times S$
- Purpose: pairs rows from two tables
- Output: for each row r in R and each s in S, output a row rs (concatenation of r and s)

Cross product example

$$User \times Member$$

uid	name	age	pop
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

uid	gid
123	gov
857	abc
857	gov
…	…



uid	name	age	pop	uid	gid
123	Milhouse	10	0.2	123	gov
123	Milhouse	10	0.2	857	abc
123	Milhouse	10	0.2	857	gov
857	Lisa	8	0.7	123	gov
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

A note a column ordering

- Ordering of columns is unimportant as far as contents are concerned

uid	name	age	pop	uid	gid
123	Milhouse	10	0.2	123	gov
123	Milhouse	10	0.2	857	abc
123	Milhouse	10	0.2	857	gov
857	Lisa	8	0.7	123	gov
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

=

uid	gid	uid	name	age	pop
123	gov	123	Milhouse	10	0.2
857	abc	123	Milhouse	10	0.2
857	gov	123	Milhouse	10	0.2
123	gov	857	Lisa	8	0.7
857	abc	857	Lisa	8	0.7
857	gov	857	Lisa	8	0.7
…	…	…	…	…	…

- So cross product is commutative, i.e., for any R and S, $R \times S = S \times R$ (up to the ordering of columns)

Derived operator: join

(A.k.a. “theta-join”)

- Input: two tables R and S
- Notation: $R \bowtie_p S$
  - p is called a join condition (or predicate)
- Purpose: relate rows from two tables according to some criteria
- Output: for each row r in R and each row s in S, output a row rs if r and s satisfy p
- Shorthand for $\sigma_p(R \times S)$

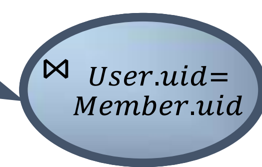
Join example

- Info about users, plus IDs of their groups

$$User \bowtie_{User.uid=Member.uid} Member$$

uid	name	age	pop
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

uid	gid
123	gov
857	abc
857	gov
…	…



uid	name	age	pop	uid	gid
123	Milhouse	10	0.2	123	gov
[redacted]					
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

Prefix a column reference with table name and "." to disambiguate identically named columns from different tables

Derived operator: natural join

- Input: two tables R and S
- Notation: $R \bowtie S$
- Purpose: relate rows from two tables, and
  - Enforce equality between identically named columns
  - Eliminate one copy of identically named columns
- Shorthand for $\pi_L\left(R \bowtie_p S\right)$, where
  - p equates each pair of columns common to R and S
  - L is the union of column names from R and S (with duplicate columns removed)

Natural join example

$$User \bowtie Member = \pi_{?}(User \bowtie_{?} Member)$$
$$= \pi_{uid,name,age,pop,gid}\left(User \bowtie_{\substack{User.uid= \\ Member.uid}} Member\right)$$

uid	name	age	pop
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

uid	gid
123	gov
857	abc
857	gov
…	…

⋈

uid	name	age	pop	[redacted]	gid
123	Milhouse	10	0.2	[redacted]	gov
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
857	Lisa	8	0.7	[redacted]	abc
857	Lisa	8	0.7	[redacted]	gov
…	…	…	…	[redacted]	…

Union

- Input: two tables R and S
- Notation: $R \cup S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows in R and all rows in S (with duplicate rows removed)

Difference

- Input: two tables R and S
- Notation: $R - S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows in R that are not in S

Derived operator: intersection

- Input: two tables R and S
- Notation: $R \cap S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows that are in both R and S
- Shorthand for $R - (R - S)$
- Also equivalent to $S - (S - R)$
- And to $R \bowtie S$

Renaming

- Input: a table R
- Notation: $\rho_S\ R$, $\rho_{(A_1,A_2,\ldots)}R$, or $\rho_{S(A_1,A_2,\ldots)}R$
- Purpose: “rename” a table and/or its columns
- Output: a table with the same rows as R, but called differently
- Used to
  - Avoid confusion caused by identical column names
  - Create identical column names for natural joins
- As with all other relational operators, it doesn’t modify the database
  - Think of the renamed table as a copy of the original

Renaming example

- IDs of users who belong to at least two groups

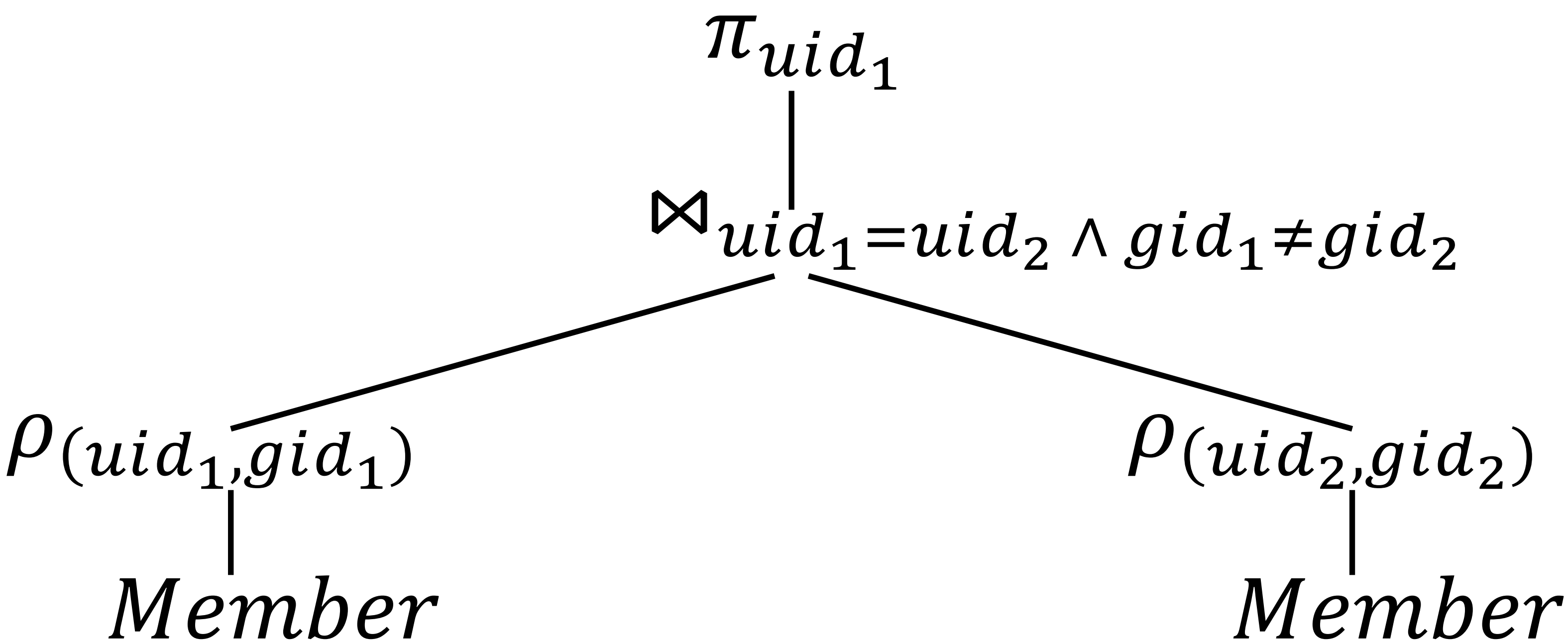
$$Member \bowtie_? Member$$

$$\pi_{uid}\left(Member \bowtie_{\substack{Member.uid=Member.uid \,\wedge \\ Member.gid \neq Member.gid}} Member\right)$$

WRONG!

$$\pi_{uid_1}\left(\begin{array}{c}\rho_{(uid_1, gid_1)} Member \\ \bowtie_{uid_1 = uid_2 \,\wedge\, gid_1 \neq gid_2} \\ \rho_{(uid_2, gid_2)} Member\end{array}\right)$$

Expression tree notation



Summary of core operators

- Selection: $\sigma_p R$
- Projection: $\pi_L R$
- Cross product: $R \times S$
- Union: $R \cup S$
- Difference: $R - S$
- Renaming: $\rho_{S(A_1, A_2, \dots)} R$
    - Does not really add "processing" power

Summary of derived operators

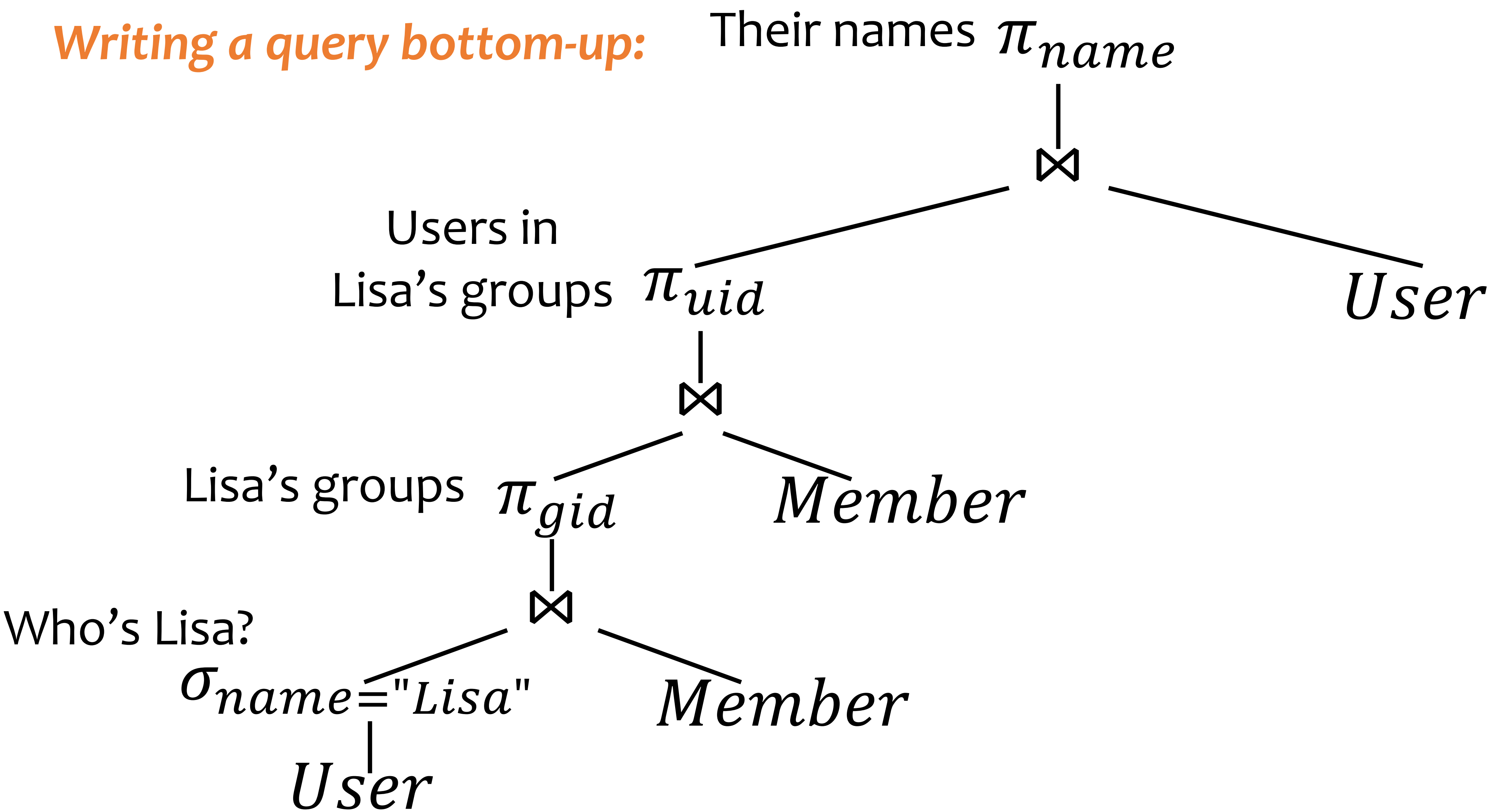
- Join: $R \bowtie_p S$
- Natural join: $R \bowtie S$
- Intersection: $R \cap S$

- Many more
  - Semijoin, anti-semijoin, quotient, …

An exercise

- Names of users in Lisa's groups

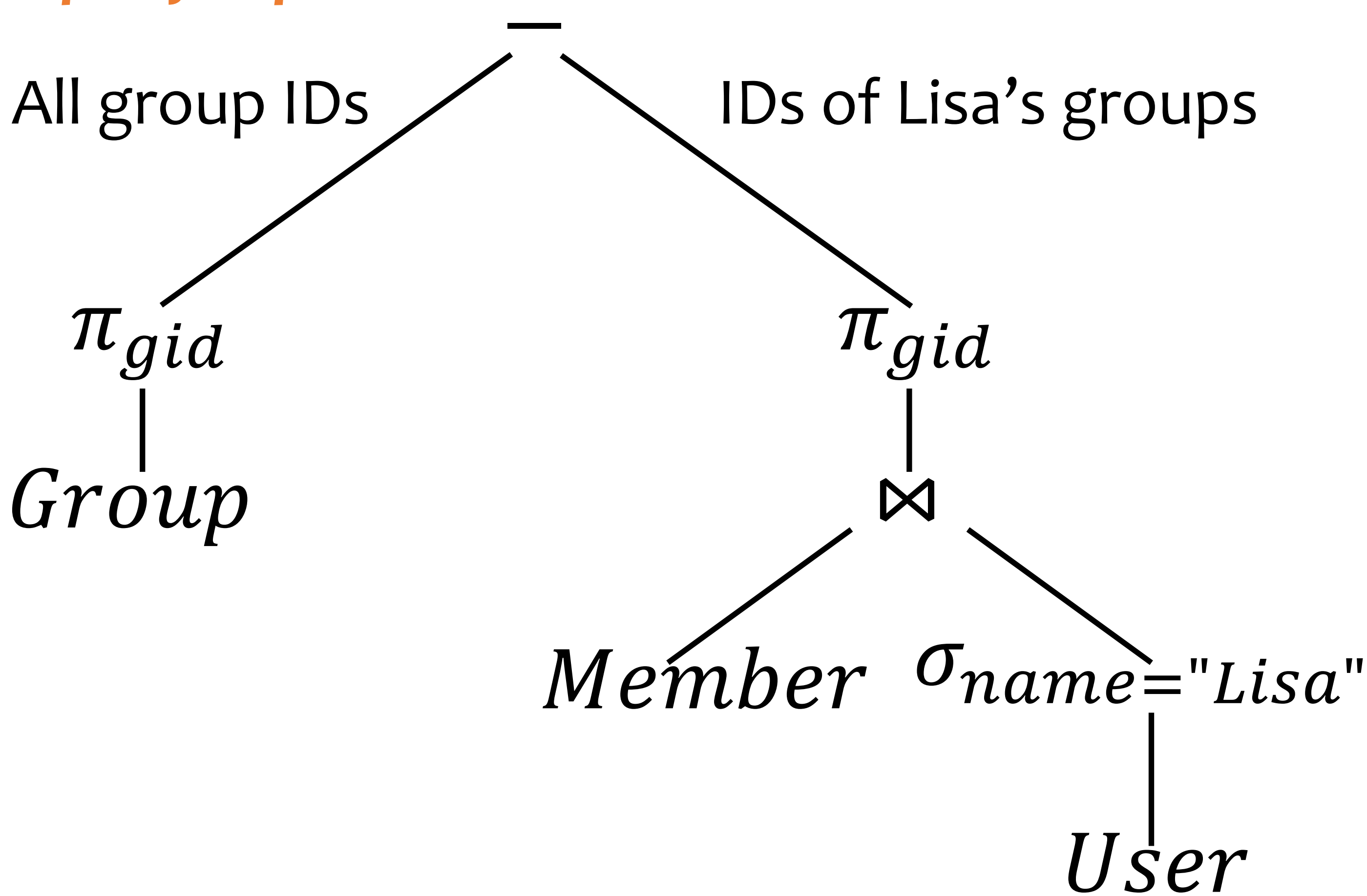
Writing a query bottom-up:



Another exercise

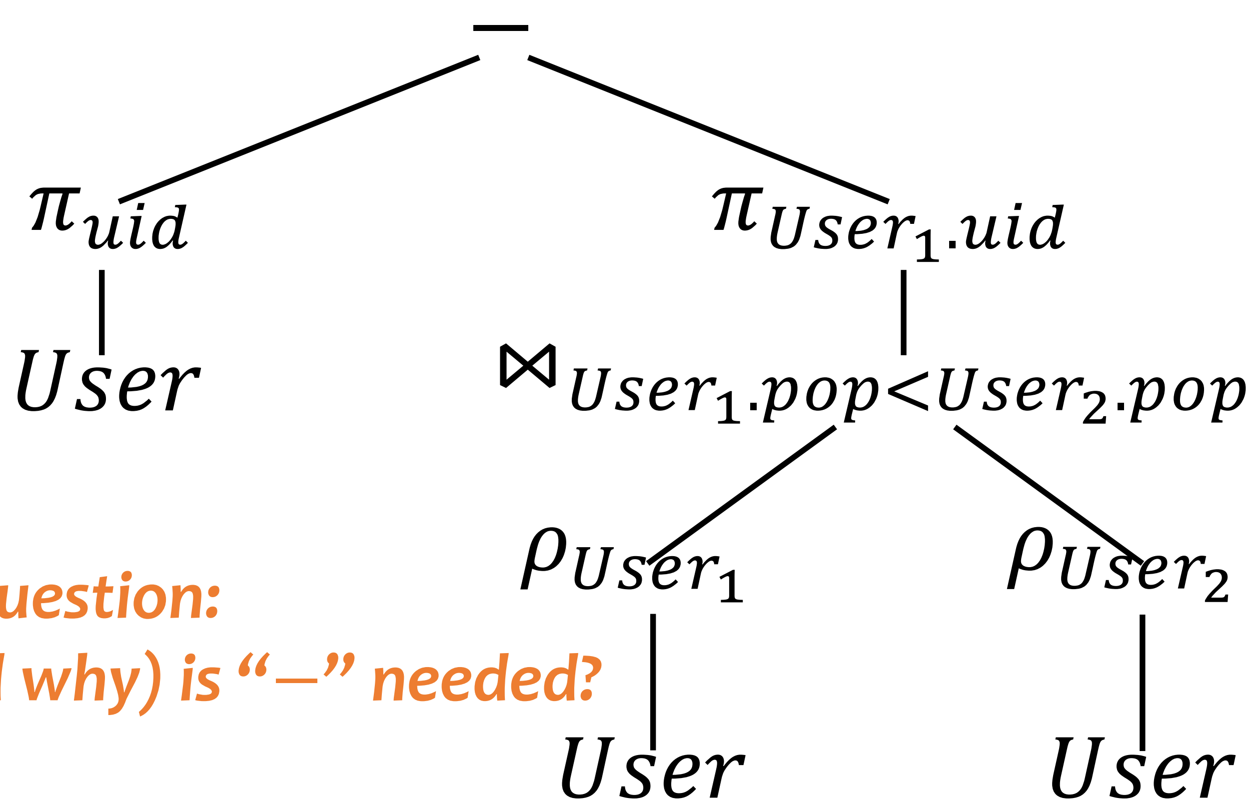
- IDs of groups that Lisa doesn't belong to

Writing a query top-down:



A trickier exercise

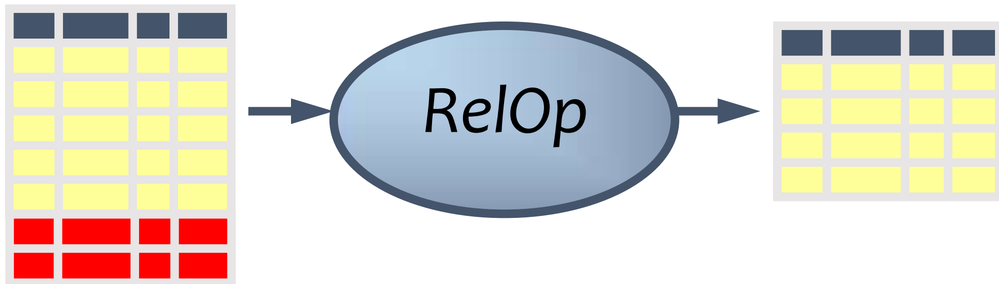
- Who are the most popular?
  - Who do NOT have the highest *pop* rating?
  - Whose *pop* is lower than somebody else's?



A deeper question:
When (and why) is "—" needed?

Monotone operators

Add more rows to the input...



What happens to the output?

- If some old output rows may need to be removed
  - Then the operator is non-monotone
- Otherwise the operator is monotone
  - That is, old output rows always remain "correct" when more rows are added to the input
- Formally, for a monotone operator op:
  $R \subseteq R'$ implies $op(R) \subseteq op(R')$ for any R, R'

Classification of relational operators

• Selection: $\sigma_p R$	Monotone
• Projection: $\pi_L R$	Monotone
• Cross product: $R \times S$	Monotone
• Join: $R \bowtie_p S$	Monotone
• Natural join: $R \bowtie S$	Monotone
• Union: $R \cup S$	Monotone
• Difference: $R - S$	Monotone w.r.t. R; non-monotone w.r.t S
• Intersection: $R \cap S$	Monotone

Why is “—” needed for “highest”?

- Composition of monotone operators produces a monotone query
  - Old output rows remain “correct” when more rows are added to the input
- Is the “highest” query monotone?
  - No!
  - Current highest *pop* is 0.9
  - Add another row with *pop* 0.91
  - Old answer is invalidated

☞So it must use difference!

Why do we need core operator X?

- Difference
  - The only non-monotone operator
- Projection
  - The only operator that removes columns
- Cross product
  - The only operator that adds columns
- Union
  - The only operator that allows you to add rows?
  - A more rigorous argument?
- Selection?
  - Homework problem

Extensions to relational algebra

- Duplicate handling (“bag algebra”)
- Grouping and aggregation
- “Extension” (or “extended projection”) to allow new column values to be computed

☞All these will come up when we talk about SQL

☞But for now we will stick to standard relational algebra without these extensions

Why is r.a. a good query language?

- Simple
  - A small set of core operators
  - Semantics are easy to grasp
- Declarative?
  - Yes, compared with older languages like CODASYL
  - Though operators do look somewhat "procedural"
- Complete?
  - With respect to what?

Relational calculus

- $\{u.uid \mid u \in User \land \neg(\exists u' \in User: u.pop < u'.pop)\}$, or
- $\{u.uid \mid u \in User \land (\forall u' \in User: u.pop \geq u'.pop)\}$
- Relational algebra = "safe" relational calculus
  - Every query expressible as a safe relational calculus query is also expressible as a relational algebra query
  - And vice versa
- Example of an "unsafe" relational calculus query
  - $\{u.name \mid \neg(u \in User)\}$
  - Cannot evaluate it just by looking at the database

Turing machine

- A conceptual device that can execute any computer algorithm
- Approximates what general-purpose programming languages can do
  - E.g., Python, Java, C++, …

Alan Turing (1912-1954)

☞So how does relational algebra compare with a Turing machine?

http://en.wikipedia.org/wiki/File:Alan_Turing_photo.jpg

Limits of relational algebra

- Relational algebra has no recursion
  - Example: given relation *Friend*(*uid1*, *uid2*), who can Bart reach in his social network with any number of hops?
    - Writing this query in r.a. is impossible!
  - So r.a. is not as powerful as general-purpose languages
- But why not?
  - Optimization becomes undecidable

  ☞Simplicity is empowering
  - Besides, you can always implement it at the application level, and recursion is added to SQL nevertheless!